State-Estimation Techniques for a Simple 3DOF Structure

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I. Abstract

Structural health monitoring (SHM) is a relatively new field of study in structural engineering, with the goal of either measuring or predicting the value of different states (e.g. displacement, strain) in a structure. The measured or estimated states of the structure are then used to determine if the structure has remained in its domain of safe operation, even through major events such as earthquakes and blast loads. This allows for rapid damage assessment after major events or in regular use.

This project will use state estimation techniques from control theory (e.g. Kalman filtering), combing system models with sensor data, in an attempt to monitor as many states as possible given limited sensor data for a simple structural frame.

II. Introduction

I. Motivation & Background

Traditionally, measurement of structural states only occurred during periodic inspections of structures and was limited to temporally discrete measurements of a subset of the structural components. With modern day wired and wireless sensor networks, however, this is changing and the continuous monitoring of structural starts with the installation of sensors and continues to the present time. However, practically there is still a limit to the number of these sensors which can be placed on a structure for various reasons (e.g. cost, accessibility).

Ideally every state (e.g.displacement, strain, acceleration), or at least a large subset of all possible building states, would be measured at all times through a structure's life. These measurements would be used to ensure each respective state stayed within the bounds of the building or structure's design and operation specification.

Thus, there exists a need to be able to know as many time histories of states of a structure as possible, yet instrument as few of the states as possible. This problem lends itself well to state-estimation techniques combining sensor data and system mathematical models, for example the Kalman Filter.

The authors have a background in structural engineering, as well as sensors and sensor networks, making them especially qualified and interested in this topic.

II. Relevant Literature

Building structures and ensuring they survive dynamic loading events (e.g. earthquakes) has been a concern of humans since the dawn of time. From a desire to predict how a proposed structure will respond to a loading event, the field of structural dynamics has developed many analysis techniques using mathematical models to "pre-evaluate" the performance of structural designs [1, 2]. These models are now widely used in building design for seismic zones.

Now, consider a building designed using these dynamic models *after* some major event like an earthquake. In theory, the models predicted faithfully how the building actually behaved, and thus it is safe to resume normal operations. However, the only way to confirm this is to inspect the building post dynamic loading. In an answer to the need to understand the states of the building (e.g. inter-story drift) during service-life, structural health monitoring (SHM) [3] has been growing in popularity. SHM consists of using sensor networks, wired or wireless, to monitor building states during service-life. This monitored data can then be used to rapidly evaluate if the structure is safe after a dynamic event.

With the ability to now monitor building states during the service-life of structures, and use this data to determine the health of the structure, the question now becomes, "What states should be instrumented?" The short answer is all states, thus engineers can be sure about the building's behavior during a loading event. However, instrumentation limitations, such as cost or accessibility, limit the number of states that can be actually instrumented. This begs the question, given these limited sensor data and the mathematical dynamic structural model of the structure, can state estimation techniques (e.g. Kalman Filtering) be used to *estimate* the states of the building which weren't actually instrumented?

III. Focus of this Study

Given limited sensor placements for structural states (perhaps from a wireless sensor network), and the existence of literally centuries of validated structural models, structural health monitoring (SHM) is a field ripe for state estimation techniques. This study will focus on a planner-symmetric, shear-wall building, consisting of three floors (assumed to be rigid diaphragms) which can be modeled as a simple N degree-of-freedom (N DOF) structural system. For simplicity, the structure will be excited in one direction, perpendicular to its strong axis of bending. The goal of the study will be to measure and/or estimate the lateral position of each floor throughout a dynamic loading event, while only instrumenting a subset of the floors.

III. Technical Description

I. Approach

A N degree-of-freedom structural system system, see Figure 1 with N=3, will be considered. A first principles mathematical model will first be constructed of this system using typical parameter values. This model, along with a set of typical ground motions, will be used to create training sets of data consisting of building states with respect to certain ground motions. Synthetic noise will be added to these state values and they will be assumed to be measured data from a real structure. With varying subsets of the synthetic sensor data sets, the corresponding ground motions, and the original first principles model, a state estimation framework will be constructed. The goal will be to reduce the available synthetic sensor data streams to the smallest set possible (recall each stream corresponds to a sensor node), while still maintaining the ability to estimate each structural state. Given the total set of displacement is actually known, rigorous metrics defining error can be used to evaluate the state estimation framework.

II. Structural Dynamics Mathematical Model

A planer-symmetric building, with rigid floor plates, excited in a direction perpendicular to the strong flexural axis, can be modeled as a N degree-of-freedom mass-spring-damper system, like the N=3 example in Figure 1. Assuming our model will stay within the linear regime of excitement (a large assumption given earthquake loading, however, needed due to course project limitations), the equations of motion based on the Newton's Second Law presented in canonical form are,

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{j}\ddot{u}_g(t) \tag{1}$$

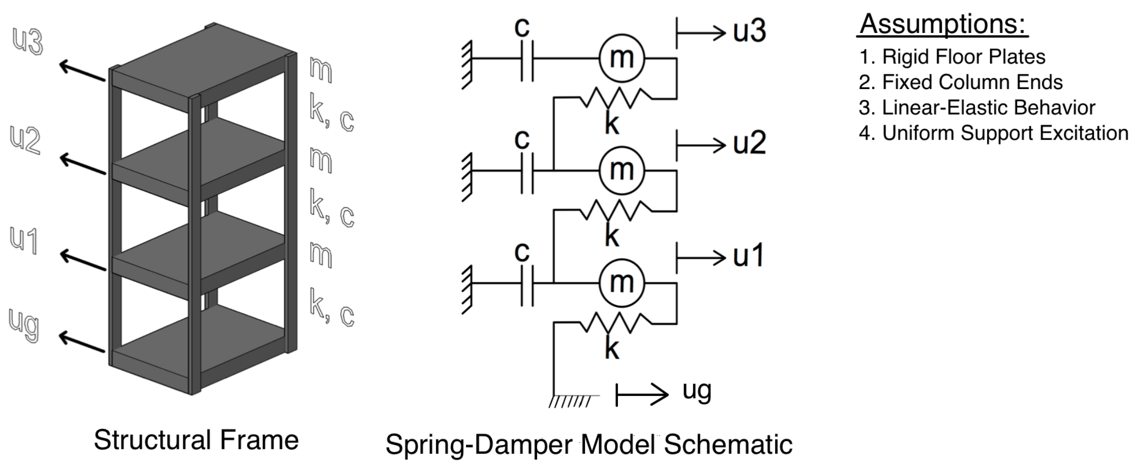

Figure 1: Diagram of actual structural model (left) Spring-Damper Dynamic Structural Model formulation (right)

The states in our system will be lateral position described by the vector $\mathbf{u}$ such that,

$$\mathbf{u} = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}. \tag{2}$$

The parameters of the system, include the mass of each floor ($\mathbf{m}$), the stiffness of the columns ($\mathbf{k}$) -all assumed identical-, and the damping matrix ($\mathbf{c}$), all defined as:

$$\mathbf{m} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \mathbf{k} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}. \tag{3}$$

k being the stiffness of any given story defined as,

$$k = \sum_{columns} \frac{12EI_c}{h^3}, \tag{4}$$

E as Younge's Modulus, I_c as the second moment of inertia around the bending axis, and h as the inter-story height. $\mathbf{j}$ is a spatial distribution vector equal to unity because the axis of motion is aligned with the degrees-of-freedom. $\ddot{u}_g(t)$ is a parameter as well, however, not of the model, but the analysis being conducted. $\ddot{u}_g(t)$ is defined as the ground motion acceleration measured by a strong-motion seismograph during actual earthquake events. For example, it is common to use more "famous" earthquake ground motions, for example the 1995 Kobe, Japan earthquake or the 1994 Northridge, CA, USA earthquake.

$\mathbf{m}$ and $\mathbf{k}$ can be calculated from the structure being modeled, and $\ddot{u}_g(t)$ can be acquired from a database such as Pacific Earthquake Engineering Research Center (PEER) Ground Motion Database. $\mathbf{c}$, however, is impractical to calculate from structural properties and real data is very limited from which to identify it, thus it is standard practice in the industry to estimate $\mathbf{c}$ given one of several methods, with the Caughey method covered in chapter 11 of [1] used here.

II.1 State-Space Model Formulation

Given the canonical form of the equation of motion, to use the simulation tools explored in CE295 the model will need to be converted into canonical linear state-space system form. In order to accomplish this, another N state variables will need to be added due to the second order nature of each state equation. In continuing the states from Figure 1, this changes our states to,

$$\mathbf{u} = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \dot{u}_1(t) \\ \dot{u}_2(t) \\ \dot{u}_3(t) \end{bmatrix}. \tag{5}$$

Now, the $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ matries must be constructed, done so here in the more abstract N degree of freedom notation. For $\mathbf{A}$ and $\mathbf{B}$, the damping and stiffness terms (i.e. $\mathbf{c}$, $\mathbf{k}$) must be subtracted to the excitation side of the equation, then each equation is divided by mass. Notice, this can accomplished via the following block matrix compositions:

$$\mathbf{A}_{lowerLeft} = -\mathbf{m}^{-1}\mathbf{k}, \quad \mathbf{A}_{lowerRight} = -\mathbf{m}^{-1}\mathbf{c}, \tag{6}$$

with,

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{N,N} & \mathbf{I}_{N,N} \\ \mathbf{A}_{lowerLeft} & \mathbf{A}_{lowerRight} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{N,1} \\ \mathbf{1}_{N,1} \end{bmatrix}. \tag{7}$$

Considering the construction of $\mathbf{C}$, the sensor type and placement location on the structure must known. If velocity or position is being measured on a certain floor, a corresponding one should be placed to output that value from the state vector. If acceleration is being measured, a one should be placed in the corresponding velocity position. The measured acceleration will need to be integrated to be used as it's not a state variable, but rather the derivative of the velocity state variable. A sample $\mathbf{C}$ which corresponds to floor one displacement being measured is,

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8}$$

Given that sensor placement is critical to the observability of the system, a discussion of how to construct $\mathbf{C}$ intelligently is presented in section III.1, System Observability. A sensor placement for the first floor is used here because measuring displacement of the first floor with respect to the base is something that can practically implemented given today's technology. While accelerator integration techniques exist, direct displacement measurement is still more accurate.

Matrix $\mathbf{D} = \mathbf{0}$ as the ground acceleration input to the structure is not needed for the analysis.

II.2 Simulation Results

In order to implement a state-estimation framework, data is needed for a structural system excited by earthquake ground motion. For this investigation, a simulation using the 1995 Kobe, Japan earthquake at "full-strength" was conducted. The results of which can be seen in Figure 2. The model parameters, that is story lumped mass and stiffness where calculated using structural engineering judgment and reference to the American Institute of Steel Construction's, "Steel Construction Manual". The details are beyond the scope of this study.

Further, to simulate the inevitable differences between the observer/estimator model and the actual system model, random noise of approximately 0.1% was added to the system matrices before simulation. The non-noise induced matrices were then used in the actual observer/estimator frameworks presented below.

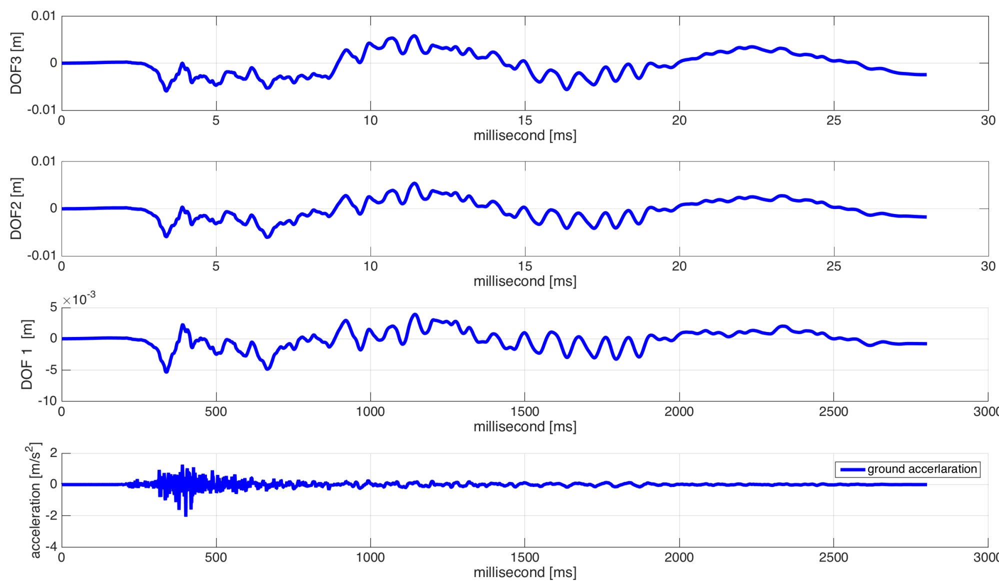

Figure 2: Plots of the raw simulation results for degree of freedom one through three displacements in meters and ground acceleration.

III. State-Estimation Techniques

In this experiment, two state estimation techniques are used (i.e. Luenberger Observer, Kalman Filter) to estimate all state variables given limited state measurements. The simulation described above produces the trajectories of *all states* for the entire time horizon of the simulation. By examining various subsets of the state trajectories time histories, adding mock "sensor noise" and plugging them into a state estimation framework, the remaining simulated results can be used to quantify accuracy of the state estimation technique being investigated.

To begin the investigation, observability of the system is explored.

III.1 System Observability

Given some state-space system with n states, consider the observability matrix,

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}. \tag{9}$$

If the column rank of $\mathcal{O}$ is n, the matrix is *observable*, and all states can be estimated effectively. If it is less than n, the system is only *partially oberservable* or *unobservable*. The Observability matrix is accessing the “connectedness” of the various states within our model. Hence, the A^{n-1} is used to check whether the states that are being observed through the C matrix are impacted by other states and hence we can “see” this impact through measurements of C thus giving us an ability to estimate the state.

Given the system matrices A, C described above we can see sensor placement on floor one of a three floor building yields a fully observable system because its Observability matrix has full column rank. Imaging the Mass-Spring-Damper model being used, however, intuition lends the understanding that regardless of the floor sensor placement the system will always be fully observable.

To better see this, image any single story of the model being displaced one unit, while the others are held constant, then the displaced floor and all other floors are released at the same instant. Its clear all the floors (or masses in our model) will be displaced in some manner until all the energy has dissipated through damping. Like a slinky on a table will move with “wave-like” patterns if excited at one end. This intuition, that all the masses are connected in their motion, offers an insight into why the structure will always be fully observable, even with the limits of one sensor.

III.2 Luenberger Observer

This observer is based on the notion of introducing the error in measurement/prediction into the state estimation dynamics. It’s particular power is in the ability to control the error dynamics via pole placement to be faster than the system dynamics, thus improving on a Open-Loop Observer. Given standard state space estimation notion it takes the from:

$$\dot{\hat{x}} = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)], \quad \hat{x}(0) = \hat{x}_o \quad \hat{y}(t) = C\hat{x}(t) + Du(t) \tag{10}$$

When applied to the three floor structural system examined here, we can see the state estimation results presented in Figure 3. To simulate sensor noise, an ever present component in real systems, a normally distributed noise value was added to the output value of the first floor. The noise has mean 0 and standard deviation equal to 10% of the maximum displacement of DOF 1.

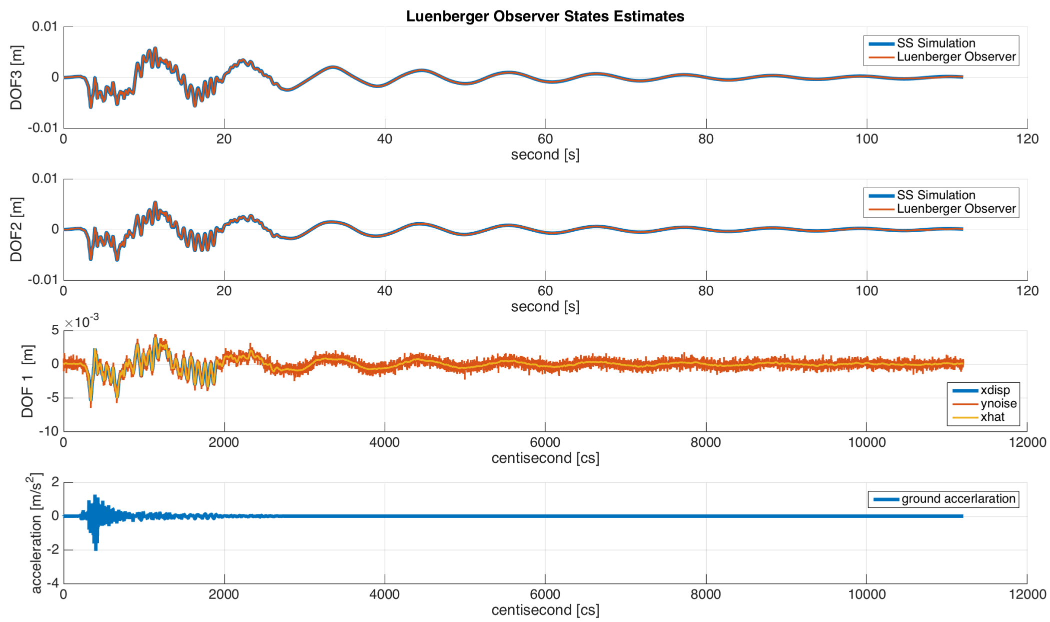

Figure 3: Luenberger Observer state estimation results.

Further, a zoomed in plot for the first floor can be seen in Figure 4.

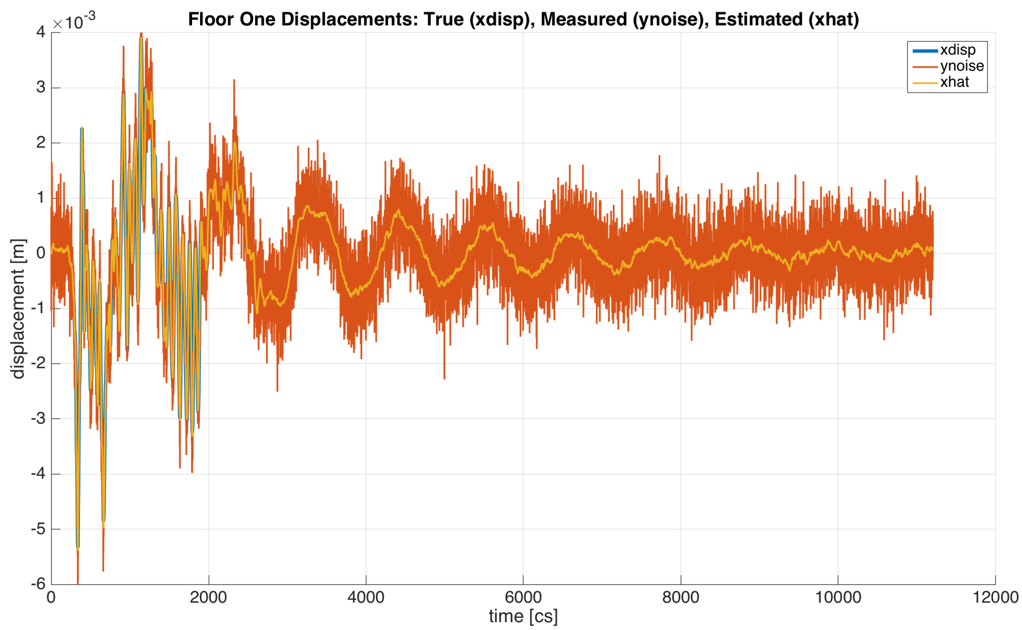

Figure 4: Luenberger Observer state estimation results for DOF 1 magnified.

III.3 Kalman Filter

Similar to the Luenberger Observer, the Kalman filter aims to estimates states as a balance between model simulation and sensor measurements. Within the assumptions of the framework, the estimates are optimal and will take the form:

$$\dot{\hat{x}} = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)], \quad \hat{x}(0) = \hat{x}_o \quad \hat{y}(t) = C\hat{x}(t) + Du(t), \tag{11}$$

where,

$$L(t) = \Sigma(t)C^T G^{-1} \tag{12}$$

$$\dot{\Sigma}(t) = \Sigma(t)A^T + A\Sigma(t) + W - \Sigma(t)C^T G^{-1} C\Sigma(t), \quad \Sigma(0) = \Sigma_o \tag{13}$$

where Σ is the solution to the differential Equation 13 and G being sensor and W being process noise covarience matrices respectively.

The results of the Kalman Filter applied to the same three degree of freedom system investigated here can be seen in Figure 5. Again, the same sensor noise used for the Luenberger Observer is used here on the sensor output used for state estimation.

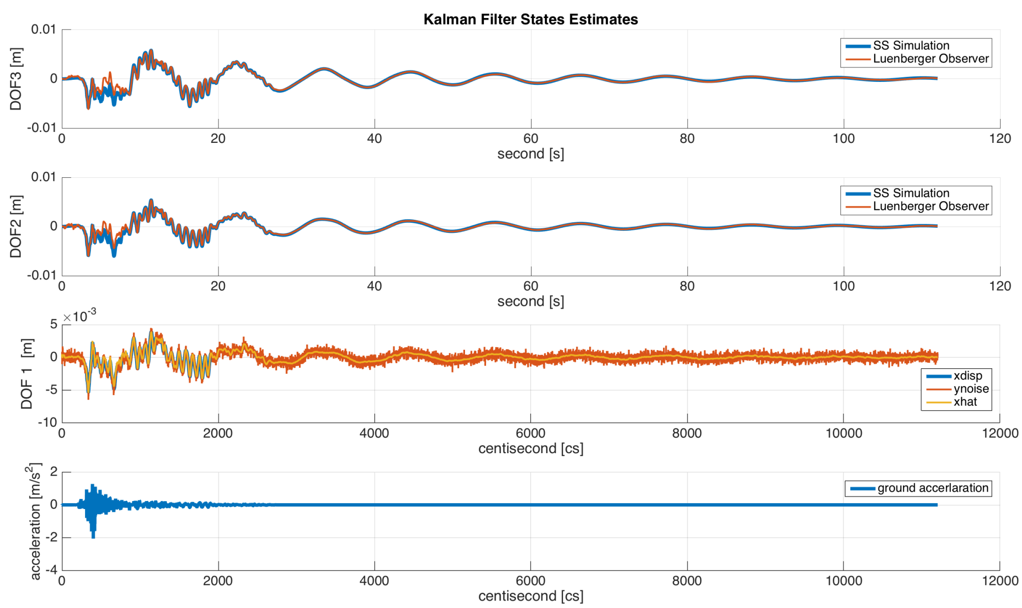

Figure 5: Kalman Filter state estimation results.

Again, a zoomed in plot for the first floor can be seen in Figure 6. Further, the Kalman Filter provides a bound of plus/minus one standard deviation of the state estimate (recall the Kalman Filter is a stochastic estimator). These bounds are also included in the plot as the dashed lines above and below the states estimate trajectory.

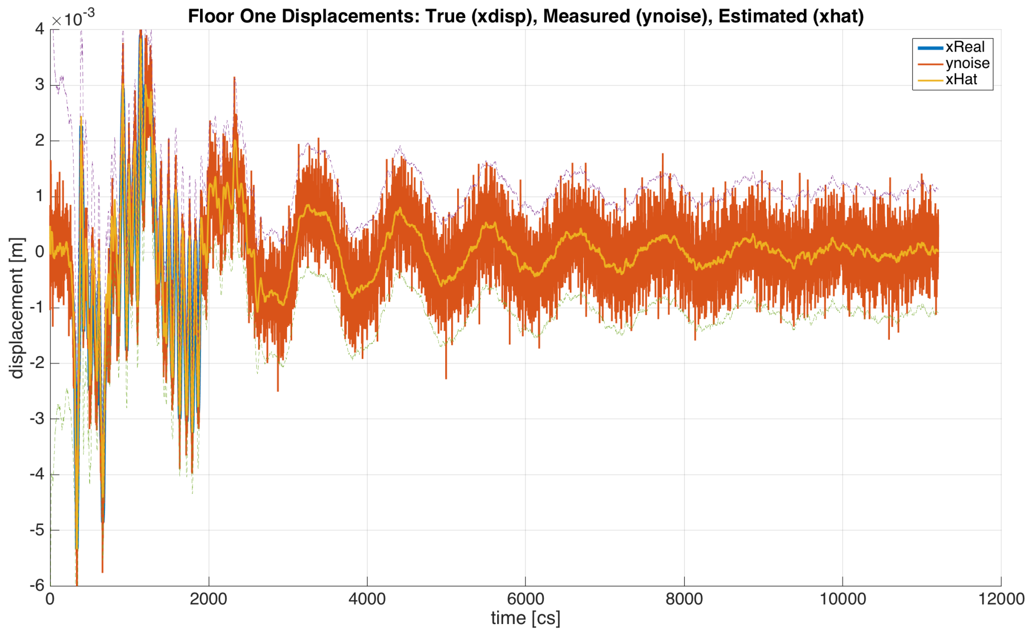

Figure 6: Kalman Filter state estimation results for DOF 1 magnified. The faint dashed lines, while not labeled in the legend, are the one standard deviation bound of the state estimate.

IV. Discussion

I. Luenberger Observer

Qualitatively the Luenberger observer does a great job at estimating the states of three degrees of freedom (i.e. the three floors of the building) of the system under study. Examining Figure 4, as magnified section of the motion for DOF 1, specifically it can be seen that even with 10% sensor noise the state estimate is very close to the true state value.

In this system, the error value is almost constant throughout the simulation because the initial conditions are correct in the observer because the building is at rest at the beginning of the seismic event. Thus, there is no convergence of the estimation as it is more typically observed in state estimation activities. Also, it should be noted, typically normalized error is used, however, due to the state variables being around zero, there exists somewhat random blow-up of the normalized error near displacement is zero in this case. As such, simply total error is presented, see Figure 7. Notice, however, the order of magnitudes of the absolute error, as compared to the state values, and one can deduce it is quite accurate.

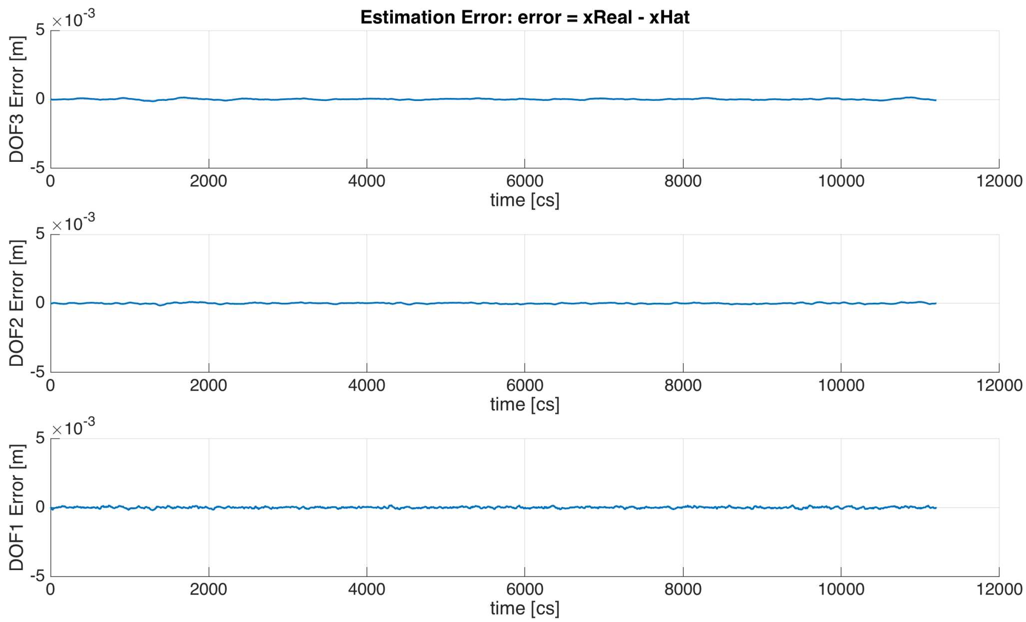

Figure 7: Luenberger Observer absolute state estimation error.

II. Kalman Filter

In Figure 5 we can see the Kalman filter has some regions where it is very effective at estimating the state of the system, while in others, there are noticeable differences. This is characterized explicitly in Figure 8. While the authors are unsure exactly why this is the case, they hypothesize that the stochastic nature of the Kalman Filter estimation framework causes a slight lag in the sensor reading impacts on the state estimation. This idea is taken from the fact that the larger errors come immediately after what are relatively rapid changes in state of the system. The reason these changes take a little longer to impact the state estimate is because the Kalman framework weights these rapid states changes delivered by the sensor readings with a stochastic model, hence they are discounted. As compared to the Luenberger Observer where they are directly feed into the state estimate.

Given this system, it seems the benefits of the stochastic approach are note realized as compared to the deterministic approach. Perhaps in systems where the model in the observer/estimator is less accurate and the stochastic approach will see improved performance.

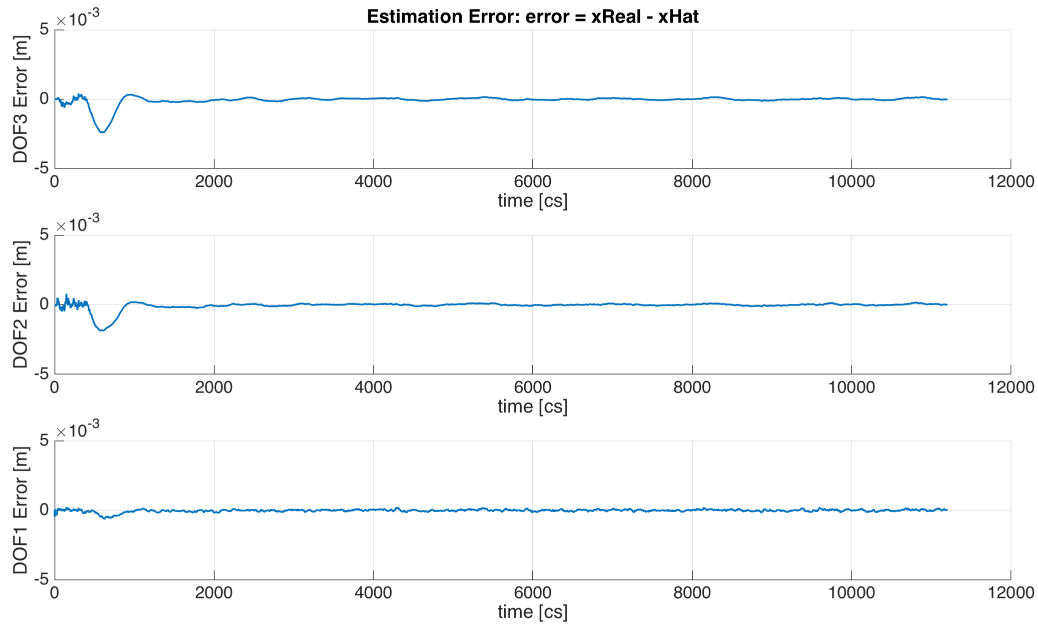

Figure 8: Kalman Filter absolute state estimation error.

III. Concluding Remarks

In conclusion, given the relatively simple model and rather deterministic physics, the Luenberger Observer performs better than the Kalman Filter given the Kobe, Japan 1995 ground motion investigated. It is hypothesized this difference is due to the lag of the Kalman Filter to rapid state changes. Perhaps in systems where the model in the observer/estimator is less accurate and the stochastic approach will see improved performance.

V. Summary

In summary, the authors have shown that even with very limited sensor placement (that is one sensor for the linear mass-spring-damper structural dynamics model) accurate state estimations can be done on civil structures for health monitoring purposes during earthquake events. While both the Luenberger Observer and Kalman Filter frameworks for estimating the system states were effective, it was shown in this example that the Luenberger Observer outperformed the Kalman Filter.

References

[1] A. K. Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Englewood Cliffs, New Jersey: Prentice Hall, 4 ed., 2012.

[2] A. K. Chopra, *Earthquake Dynamics of Structures, a Primer*. Oakland, California: Earthquake Engineering Research Institute, 2 ed., 2005.

[3] A. G. Daniel Balageas, Claus-Peter Fritzen, *Structural Health Monitoring*, vol. 493. Wiley-ISTE, 2006.